Continuous Preparation of a Fractional Chern Insulator

M. Barkeshli,[1] N. Y. Yao,[2] and C. R. Laumann[3,*]

[1]Microsoft Station Q, Elings Hall, University of California, Santa Barbara, Santa Barbara, California 93106, USA
[2]Department of Physics, University of California, Berkeley, Berkeley, California 94720, USA
[3]Department of Physics, University of Washington, Seattle, Washington 98195, USA

(Received 10 November 2014; revised manuscript received 27 May 2015; published 7 July 2015)

We present evidence of a direct, continuous quantum phase transition between a Bose superfluid and the $\nu = 1/2$ fractional Chern insulator in a microscopic lattice model. In the process, we develop a detailed field theoretic description of this transition in terms of the low energy vortex dynamics. The theory explicitly accounts for the structure of lattice symmetries and predicts a Landau forbidden transition that is protected by inversion. That the transition is continuous enables the quasiadiabatic preparation of the fractional Chern insulator in nonequilibrium, quantum optical systems.

DOI: 10.1103/PhysRevLett.115.026802 PACS numbers: 73.43.Cd, 05.30.Jp, 37.10.Jk, 73.43.Nq

The canonical examples of topological order are provided by the fractional quantum Hall states, conventionally found in two-dimensional electron gases [1,2]. Their lattice cousins, the fractional Chern insulators (FCIs), naturally arise when strongly interacting particles inhabit flat, topological band structures [3–17]. Effective microscopic Hamiltonians whose ground states realize such phases have been numerically identified in synthetic quantum systems, ranging from ultracold gases in optical lattices to ensembles of solid-state defects [18–20]. On the experimental front, Aidelsburger *et al.* [21] have recently loaded ^{87}Rb into the topological, nearly-flat band of a Hofstadter model.

Unlike typical condensed matter systems, quantum optical proposals for topological phases represent driven, nonequilibrium implementations in an effective Hamiltonian picture. Thus, even if an appropriate Hamiltonian can be realized, guiding the system to its ground state is still a major challenge. Often, one cannot simply "cool" by decreasing the temperature of a surrounding bath. One approach to this problem is provided by quasiadiabatic preparation, wherein the correlated ground state is reached from a simple initial state by slowly tuning the Hamiltonian parameters. In the case of FCIs, natural starting states include superfluids (SFs) and charge-density wave (CDW) insulators, as these often arise in close proximity to the FCI state of interest (Fig. 1, [18,20]).

Quasiadiabatic preparation requires that any quantum phase transition between the initial and final state be continuous. A system tuned through a first order transition would need to be ramped exponentially slowly in system size to avoid being stuck in a metastable high energy state [22,23]. On the other hand, continuous quantum phase transitions allow for two possibilities: (1) strictly adiabatic preparation with ramp time scaling as a power law in system size [24–26] or (2) quasiadiabatic preparation with a final state energy density scaling as an inverse power law with the ramp time [27–29]. Unfortunately, there is relatively little known regarding quantum phase transitions between conventional and fractional phases as such transitions lie beyond the Ginzburg-Landau paradigm [30].

Field theories of possible critical points between Laughlin fractional quantum Hall states and Mott insulators were studied in Refs. [31–33]. Meanwhile, a theory of a superfluid to bosonic $\nu = 1/2$ Laughlin state was recently constructed in Ref. [34]. All of these theories assume that any additional lattice symmetries are preserved throughout the phase diagram. They require the bosons to be at integer filling and cannot describe CDW order. Moreover, to date, none of these continuous transitions has been established in any microscopic model, as second order phase transitions

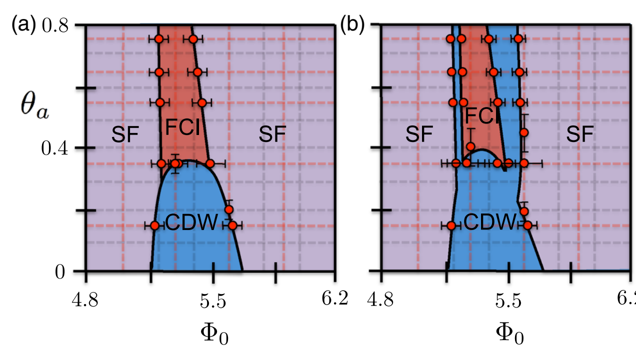

FIG. 1 (color online). (a) Two parameter phase diagram of the driven NV model as determined by exact diagonalization of Eq. (2). (b) Phase diagram in the presence of microscopic inversion symmetry breaking parameter $g = 0.2$. The (π, π) CDW insulator extends in two fingers which split the SF↔FCI transition, showing that the underlying transition at $g = 0$ is continuous and protected by inversion symmetry. Spectra and structure factors collected on coarse grey grid sites; full diagnostics (see text) calculated on 1D (red) cuts at a refined spacing of 0.01. Markers with error bars indicate regions where diagnostics were ambiguous. Markers without error bars indicate ambiguous regions narrower than marker size.

 © 2015 American Physical Society

are difficult to characterize in the small systems amenable to numerical study.

In this Letter, we report two main advances. First, we establish the presence of a direct continuous transition between a superfluid and a $\nu = 1/2$ FCI state in a microscopic model of interacting spins. We do this by showing that the direct superfluid-FCI transition splits into two transitions when we perturbatively break inversion symmetry. Since first order phase transitions are insensitive to perturbations, the splitting of the transition implies that it must be continuous. This qualitative signature avoids the usual difficulty associated with finite-size scaling in small systems. Second, we develop a detailed field theoretic description of this transition in terms of the low-energy vortex fields. This description naturally accommodates the spontaneous breaking of lattice symmetry in the Mott-insulating CDW state at half-filling.

Microscopic model.—We study the microscopic phase diagram of a two-dimensional square lattice of nitrogen-vacancy (NV) defects in diamond [35–37]. Our model is closely related to previous proposals for realizing FCI states in ultracold polar molecules [18]. We will briefly sketch the main ingredients below (for details see the Supplemental Material [38] and Ref. [20]). Each NV constitutes a spin one ($S = 1$) electronic degree of freedom and interactions occur via the magnetic dipole-dipole interaction,

$$H_{dd} = \frac{1}{2}\sum_{i\neq j}\frac{\kappa}{R_{ij}^3}[\mathbf{S}_i\cdot\mathbf{S}_j - 3(\mathbf{S}_i\cdot\hat{\mathbf{R}}_{ij})(\mathbf{S}_j\cdot\hat{\mathbf{R}}_{ij})], \tag{1}$$

where, $\kappa = \mu_0/(4\pi)$ and $\mathbf{R}_{ij}$ connects sites i and j [36].

Hyperfine interactions between the NV electronic spin and the nitrogen nuclear spin (assuming isotope ^{15}N), lead to 6 states on each site, indexed $|S_z, I_z\rangle$, where $S_z = \pm 1, 0$, $I_z = \pm 1/2$. Taking into account the zero-field splitting as well as an applied magnetic field, it is possible to arrange for the low-energy manifold on each site to contain only 4 states, namely, the $S_z = 0, +1$ states. By applying suitable optical dressing, the effective dynamics can be further restricted to a two-level system, with local dark states $|0\rangle = \beta|1,-\frac{1}{2}\rangle - \alpha|0,\frac{1}{2}\rangle$, and $|1\rangle = s|0,-\frac{1}{2}\rangle + v|1,\frac{1}{2}\rangle + w(\alpha|1,-\frac{1}{2}\rangle + \beta|0,\frac{1}{2}\rangle)$. The coefficients α, β are fixed by the hyperfine interaction and applied static fields while the coefficients s, v, w are tunable via dressing to the electronic excited state A_2 [39]. In the rotating frame, the states $|0\rangle$ and $|1\rangle$ are split by an energy Δ, which is of order the hyperfine coupling [20,38].

The characteristic magnetic dipolar interaction strength, κ/R_0^3 is typically much weaker than Δ, assuming R_0, the nearest-neighbor lattice spacing, is $\gtrsim$10 nm. Thus, while the dipolar interaction can "flip-flop" $|10\rangle \leftrightarrow |01\rangle$ between sites resonantly, processes which change the total number of 1 sites are energetically suppressed. This emergent conservation law allows us to consider the system in terms of conserved hardcore bosonic operators, $a_i^\dagger = |1\rangle\langle 0|_i$, described by the Hamiltonian

$$H_B = -\sum_{ij} t_{ij} a_i^\dagger a_j + \frac{1}{2}\sum_{i\neq j} V_{ij} n_i n_j, \tag{2}$$

where $t_{ij} = -\langle 1_i 0_j|H_{dd}|0_i 1_j\rangle$ and $V_{ij} = \langle 1_i 1_j|H_{dd}|1_i 1_j\rangle + \langle 0_i 0_j|H_{dd}|0_i 0_j\rangle - \langle 1_i 0_j|H_{dd}|1_i 0_j\rangle - \langle 0_i 1_j|H_{dd}|0_i 1_j\rangle$. In addition to boson number conservation, H_B is symmetric under lattice translations and spatial inversion, but not generically under any further lattice rotations unless the NV axis is perpendicular to the lattice plane. We note that the elliptical polarization of the optical dressing fields directly breaks time-reversal symmetry [18,38].

An FCI can be realized in this system with two main kinetic ingredients: the single boson bands ought to be "flat," such that their dispersion is small relative to the interactions, and they ought to carry a nontrivial Chern number. Such topological flat bands may be achieved by using different optical dressing parameters on the a and b sites of a two-site unit cell (green and blue, inset, Fig. 2); this amounts to defining the hardcore boson slightly differently on the a and b sublattices [16].

We now consider the many-body phases which arise at filling fraction $\nu = 1/2$ per unit cell, i.e., 1/4 particle per site, in a topological flat band regime. The phase diagram depicted in Fig. 1(a) is calculated using exact diagonalization for sizes up to $N_{\text{sites}} = 36, N_{\text{particles}} = 9$. Two microscopic parameters are varied: Φ_0 is the azimuthal angle of the NV axis relative to the lattice plane and θ_a is a microscopic dressing parameter. Roughly speaking, θ_a controls the magnitude of the effective interaction V_{ij} (with $\theta_a \to 0$ giving the strongest interactions), while Φ_0 controls the amount of band dispersion. These qualitative differences in the microscopics yield a rich phase diagram exhibiting both conventional and topological phases [Fig. 1(a)].

A $\nu = 1/2$ bosonic Laughlin FCI arises where the dispersion is flattest and the dipolar tail of the interaction is weak. Turning up the interactions by varying θ_a causes the system to spontaneously break the lattice translational symmetry and form a commensurate CDW insulator at momentum (π,π). Tuning away from the flat band regime by adjusting Φ_0 leads to a phase transition into a superfluid, consistent with the microscopics being dominated by band dispersion. We identify these phases numerically with five diagnostics: (i) ground-state degeneracy, (ii) spectral flow under magnetic flux insertion (superfluid response), (iii) real-space structure factor $\langle n(R)n(0)\rangle$, (iv) the many-body Berry curvature $\sigma_{xy} = (1/2\pi)\iint F(\theta_x,\theta_y)d\theta_x d\theta_y$ with $F(\theta_x,\theta_y) = \mathrm{Im}[\langle(\partial\Psi/\partial\theta_y)|(\partial\Psi/\partial\theta_x)\rangle - \langle(\partial\Psi/\partial\theta_x)|(\partial\Psi/\partial\theta_y)\rangle]$ [40], and (v) (for the FCI), Laughlin quasihole counting [11,38].

The above diagnostics unambiguously determine the phases deep within each phase. The phase boundaries sketched in Fig. 1(a) correspond to the regions where the diagnostics become ambiguous due to the finite size crossovers. The error bars in the phase diagram indicate the width of the crossover region as observed in the five diagnostics.

Whether the transition is continuous or first order is hard to extract directly by conventional methods from such small size numerics. So we use a trick: the known critical theories describing the direct SF↔FCI transition require a discrete symmetry, such as inversion, to protect them. Thus, if breaking inversion perturbatively in the microscopic model introduces a Mott insulator between the SF and FCI phases we can conclude that the underlying transition was continuous.

To test this, we introduce a weak staggering g to the horizontal nearest neighbor hopping, $t_{i,i+\hat{x}} \to (1+g)^{s_i} t_{i,i+\hat{x}}$, where s_i is 0 (1) on the a (b) sublattice. We have investigated the phase diagram with $g = 0.2$, 0.3, 0.4; the phase diagram with $g = 0.2$ is shown in Fig. 1(b) using the same numerical diagnostics as before (Fig. 2) [38]. The introduction of staggering indeed splits the FCI to SF transition revealing an intermediate CDW insulator. We view this as strong evidence that the transition at $g = 0$ is continuous and described by the field theory we develop below.

Field theory.—In order to capture the phase transitions seen in Fig. 1, any long-wavelength description must be able to simultaneously accommodate a $\nu = 1/2$ Laughlin state, the superfluid, and the spontaneous breaking of lattice symmetry in the CDW insulator. Previous work [34] considered the case where the Mott insulator is at integer filling and thus need not break translational symmetry.

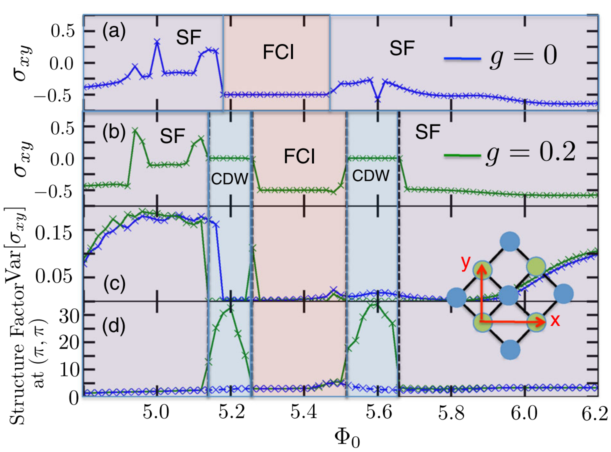

FIG. 2 (color online). Numerical diagnostics on a cut of the phase diagram at $\theta_a = 0.75$ calculated at $N_{\text{sites}} = 32$, $N_{\text{particles}} = 8$. (a) Berry curvature σ_{xy} averaged over the boundary condition torus for $g = 0$. In the SF, σ_{xy} is not quantized, while in the FCI, it is precisely -0.5. (b) Analogous with inversion broken $g = 0.2$. The intervening CDW exhibits $\sigma_{xy} = 0$. (c) Fluctuations of the Berry curvature sampled on a 10×10 grid in the boundary condition torus. Notice that fluctuations are heavily suppressed in the insulating phases while the gaplessness of the SF causes a large variance. (d) Real space structure factor at $k = (\pi, \pi)$. Both the SF and FCI are translation invariant while the CDW exhibits strong ordering. (inset) The two-site unit cell square lattice and its primitive vectors.

Here, we will present an alternate theory for bosons at half-integer filling, which takes into account the fact that the CDW insulator must *spontaneously* break lattice symmetry [41,42]. *En passant,* our new theory provides a physical representation of the transition which emphasizes the role of vortex dynamics.

We begin by briefly reviewing the effect of half-filling on the vortices of a superfluid state on a rectangular lattice [43,44]. The vortices see the original particles as magnetic flux quanta [45,46] and thus, on average, feel half a flux quantum per plaquette (of the dual lattice). This requires the translational symmetries of the vortex theory to be augmented by a gauge transformation. The resulting T_x and T_y operators satisfy the "magnetic" translation algebra $T_x T_y = -T_y T_x$. The vortex band structure must have an even number of minima, protected by this translation algebra. If these minima are not at inversion symmetric points in the magnetic Brillouin zone, then inversion symmetry $\mathcal{I}$ requires that the number of minima be a multiple of 4 [Fig. 3(a)].

In the minimal case there are four such minima at momenta $\pm k_0, \pm k_0 + (0, \pi)$ (in Landau gauge). A soft-mode expansion of the vortex field near these minima leads to four flavors of vortices which we label $\phi^v_{l\alpha}$ for $l = 0, 1$ and $\alpha = \uparrow, \downarrow$, as in Fig. 3(a). The symmetry operators act as follows:

$$\begin{aligned}\mathcal{I}&: \phi^v \to \tau_x \phi^v,\\ T_x&: \phi^v \to e^{ik_0\cdot\hat{x}\tau^z}\sigma^x\phi^v,\\ T_y&: \phi^v \to e^{ik_0\cdot\hat{y}\tau^z}\sigma^z\phi^v,\end{aligned} \tag{3}$$

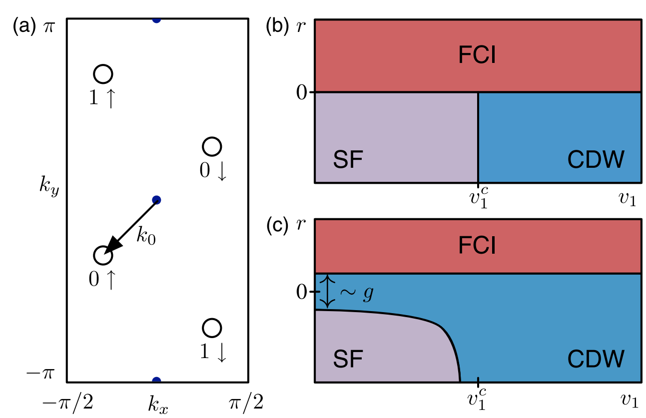

FIG. 3 (color online). (a) Magnetic Brillouin zone for vortex fields $\phi^v_{l\alpha}$ in the Landau gauge. Circles indicate dispersive minima and where the slow vortex fields are defined. (b) Two parameter phase diagram of theory defined by Eq. (4) without inversion breaking. Slice in r and v_1 holding $v_2 < v_3 < 0$, $w_2 < 0$ and $w_1, v_3, w_3 > 0$ and $u > 0$ large enough to stabilize the potential, yields an inversion breaking CDW with (π, π) ordering and a superfluid with (π, π) current order. (c) Same phase diagram with $g \neq 0$ breaking inversion.

where the τ (σ) Pauli matrices act on the α (l) index and k_0 is the momentum of the $0\uparrow$ field.

In the superfluid state, all of these vortices are uncondensed. When any combination of them condenses, the superfluid order is destroyed and the translation symmetry is broken, leading to insulating density wave states [38,43]. Remarkably, the $\nu = 1/2$ Laughlin state arises when the vortices form an integer quantum Hall state [47–51]. This motivates the following field theory which can interpolate between the FCI, superfluid, and CDW states:

$$\mathcal{L} = \frac{1}{2\pi} A_e \partial a + \frac{1}{2\pi} b^{\uparrow} \partial b^{\downarrow} - \frac{1}{2\pi} a \partial (b^{\uparrow} + b^{\downarrow}) + \sum_l |(\partial - i b \tau_x) \phi_l|^2 - V(\{\phi_l\}), \tag{4}$$

where the notation $a\partial b \equiv \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu b_\lambda$. Here, a and b^α are internal $U(1)$ gauge fields minimally coupled to the complex scalar fields $\phi_{l\alpha}$; A_e represents a background external gauge field used to probe the underlying boson current $j^\mu = 1/2\pi \epsilon^{\mu\nu\lambda} \partial_\nu a_\lambda$. The Chern-Simons terms bind a flux quantum of $b^{\uparrow/\downarrow}$ to $\phi_{l\downarrow/\uparrow}$. These flux-$\phi_{l\alpha}$ composites represent the original vortex fields $\phi^v_{l\alpha}$. Under the action of the lattice symmetries, ϕ_l can be taken to transform as ϕ^v_l in Eq. (3), while the gauge fields b are invariant under T_x, T_y and swap under $\mathcal{I}$.

The potential term $V = r\phi^\dagger\phi + V_4 + \cdots$ includes all other terms compatible with the physical and gauge symmetries. At quartic order, there are seven couplings,

$$\begin{aligned} V_4 = {} & u(\phi^\dagger\phi)^2 + v_1 \sum_l |\phi_{l\uparrow}\phi_{l\downarrow}|^2 + v_2 \sum_\alpha |\phi_{0\alpha}\phi_{1\alpha}|^2 \\ & + v_3(|\phi_{0\downarrow}\phi_{1\uparrow}|^2 + |\phi_{1\downarrow}\phi_{0\uparrow}|^2) + w_1 \sum_\alpha \phi^{*2}_{0\alpha}\phi_{1\alpha} \\ & + w_2 \phi^*_{0\uparrow}\phi_{1\uparrow}\phi^*_{0\downarrow}\phi_{1\downarrow} + w_3 \phi^*_{0\uparrow}\phi_{1\uparrow}\phi^*_{1\downarrow}\phi_{0\downarrow} + \text{c.c.} \end{aligned} \tag{5}$$

This theory, Eqs. (4) and (5), is one of the central results of the Letter. It is capable of describing all three phases found in the microscopic model: (1) When $\phi_{l\alpha}$ are uncondensed ($\langle\phi_{l\alpha}\rangle = 0$) they can be integrated out, yielding the effective theory of the $\nu = 1/2$ Laughlin state [30]. (2) If one of the $\phi_{l\alpha}$ condenses, b^α is gapped by the Anderson-Higgs mechanism; the resulting theory describes a Mott insulator which, as shown below, breaks translation symmetry. (3) If both b^α gauge fields are Higgsed, the resulting theory $\mathcal{L} = 1/(2\pi) A_e \partial a + (\partial a)^2 + \cdots$ is the usual dual description of a superfluid.

The pattern of inversion and translation symmetry breaking in these phases follows from the behavior of the simplest gauge-invariant bilinears in the ϕ fields:

$$\begin{aligned} &\mathcal{O}^\alpha_{0,0} \equiv \phi^\dagger_\alpha \phi_\alpha \quad \mathcal{O}^\alpha_{\pi,0} \equiv \phi^\dagger_\alpha \sigma^z \phi_\alpha, \\ &\mathcal{O}^\alpha_{0,\pi} \equiv \phi^\dagger_\alpha \sigma^x \phi_\alpha \quad \mathcal{O}^\alpha_{\pi,\pi} \equiv \phi^\dagger_\alpha \sigma^y \phi_\alpha. \end{aligned} \tag{6}$$

The operators $\mathcal{O}^\alpha_{k_x,k_y}$ carry momentum (k_x, k_y). The linear combination $\mathcal{O}^\pm_{k_x,k_y} \equiv \mathcal{O}^\uparrow_{kx,ky} \pm \mathcal{O}^\downarrow_{k_x,k_y}$ is inversion even (odd). Depending on which $\mathcal{O}^\pm_{k_x,k_y}$ acquire expectation values, we can determine how translation and inversion are broken [52].

Figure 3(b) shows a particular two-parameter slice of the mean-field phase diagram of Eq. (4) which shows direct continuous transitions between the FCI$\leftrightarrow$SF and FCI$\leftrightarrow$CDW phases, along with a continuous triple point terminating the first order line separating the SF$\leftrightarrow$CDW phases. The CDW order is at momentum (π, π), as seen in the numerics, while the superfluid has (π, π) current order. The leading inversion breaking potential, $V = g\phi^\dagger\tau^z\phi$, splits the direct FCI$\leftrightarrow$SF transition by an intervening CDW with width proportional to g as in Fig. 3(c). The topology of these phase diagrams matches that observed numerically in Fig. 1.

Similar phase diagrams arise in other regions of the coupling space; in all cases, the insulators exhibit commensurate density order and the SF breaks a lattice symmetry. Likewise, a superfluid living in a band structure with noninversion symmetric minima will either condense into a standing wave or break inversion. The microscopic dispersion from Eq. (2) indeed exhibits noninversion symmetric minima, but the small accessible system sizes prevent us from verifying the symmetry breaking pattern in the SF.

In summary, we have constructed a critical field theory that describes transitions between FCI$\leftrightarrow$CDW$\leftrightarrow$SF, accommodating both spontaneous symmetry breaking and topological order. Surprisingly, this theory is realized in a microscopic model of coupled electronic and nuclear spins as arise in an engineered lattice of NV defects. While our microscopic study has focused on NVs, the universal physics predicted by the field theory should be applicable to phase transitions in ultracold atomic systems [21], polar molecules [16,18], and Rydberg ensembles [53,54]. In such systems, we predict that the quasiadiabatic preparation of a fractional state can occur with energy density

$$\epsilon \sim \tau^{-\frac{3\nu}{\nu+1}} \tag{7}$$

where τ is the ramp time and ν is the correlation length exponent of the field theory [27–29]. We leave the precise calculation of ν to future work, but note that in the absence of gauge fluctuations, $\nu \approx 0.7$ [55] as for a two-component XY transition. For small finite size systems, we also expect the gap to close as $\sim 1/L$ since the dynamical critical exponent is $z = 1$. This opens the door to preparing fractionalized states in near term quantum optical simulators.

We would like to acknowledge stimulating discussions with A. Vishwanath, A. Chandran, S. D. Bennett, A. V. Gorshkov, M. D. Lukin, J. McGreevy, S. A. Parameswaran,

T. Senthil, and S. L. Sondhi. C. R. L. acknowledges the hospitality of the Perimeter Institute and the Aspen Center for Physics, supported by NSF Grant No. 1066293. N. Y. Y. acknowledges the support of the Miller Institute for Basic Research in Science.

*Corresponding author.
claumann@uw.edu

[1] D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).
[2] R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).
[3] V. Kalmeyer and R. B. Laughlin, Phys. Rev. Lett. **59**, 2095 (1987).
[4] A. Sørensen, E. Demler, and M. Lukin, Phys. Rev. Lett. **94**, 086803 (2005).
[5] M. Hafezi, A. Sørensen, E. Demler, and M. Lukin, Phys. Rev. A **76**, 023613 (2007).
[6] E. Kapit and E. Mueller, Phys. Rev. Lett. **105**, 215303 (2010).
[7] S. A. Parameswaran, R. Roy, and S. L. Sondhi, Phys. Rev. B **85**, 241308 (2012).
[8] Y.-F. Wang, Z.-C. Gu, C.-D. Gong, and D. Sheng, Phys. Rev. Lett. **107**, 146803 (2011).
[9] Z. Liu, E. J. Bergholtz, H. Fan, and A. M. Läuchli, Phys. Rev. Lett. **109**, 186805 (2012).
[10] D. N. Sheng, Z.-C. Gu, K. Sun, and L. Sheng, Nat. Commun. **2**, 389 (2011).
[11] N. Regnault and B. Bernevig, Phys. Rev. X **1**, 021014 (2011).
[12] J. McGreevy, B. Swingle, and K.-A. Tran, Phys. Rev. B **85**, 125105 (2012).
[13] K. Sun, Z. Gu, H. Katsura, and S. Das Sarma, Phys. Rev. Lett. **106**, 236803 (2011).
[14] E. Tang, J.-W. Mei, and X.-G. Wen, Phys. Rev. Lett. **106**, 236802 (2011).
[15] T. Neupert, L. Santos, C. Chamon, and C. Mudry, Phys. Rev. Lett. **106**, 236804 (2011).
[16] N. Y. Yao, C. R. Laumann, A. V. Gorshkov, S. D. Bennett, E. Demler, P. Zoller, and M. D. Lukin, Phys. Rev. Lett. **109**, 266804 (2012).
[17] G. Möller and N. R. Cooper, Phys. Rev. Lett. **108**, 045306 (2012).
[18] N. Y. Yao, A. V. Gorshkov, C. R. Laumann, A. M. Läuchli, J. Ye, and M. D. Lukin, Phys. Rev. Lett. **110**, 185302 (2013).
[19] N. R. Cooper and J. Dalibard, Phys. Rev. Lett. **110**, 185301 (2013).
[20] N. Y. Yao, S. D. Bennett, C. R. Laumann, B. L. Lev, and A. V. Gorshkov, arXiv:1505.03099.
[21] M. Aidelsburger, M. Lohse, C. Schweizer, M. Atala, J. T. Barreiro, S. Nascimbene, N. R. Cooper, I. Bloch, and N. Goldman, Nat. Phys. **11**, 162 (2015).
[22] C. R. Laumann, R. Moessner, A. Scardicchio, and S. L. Sondhi, Phys. Rev. Lett. **109**, 030502 (2012).
[23] R. Schützhold and G. Schaller, Phys. Rev. A **74**, 060304(R) (2006).
[24] J. L. Cardy, J. Phys. A **17**, L385 (1984).
[25] S. L. Sondhi, S. M. Girvin, J. P. Carini, and D. Shahar, Rev. Mod. Phys. **69**, 315 (1997).
[26] S. Sachdev, *Quantum Phase Transitions* (Cambridge University Press, Cambridge, England, 2011).
[27] T. W. B. Kibble, J. Phys. A **9**, 1387 (1976).
[28] W. H. Zurek, Nature (London) **317**, 505 (1985).
[29] A. Chandran, A. Erez, S. S. Gubser, and S. L. Sondhi, Phys. Rev. B **86**, 064304 (2012).
[30] X.-G. Wen, *Quantum Field Theory of Many-Body Systems* (Oxford University Press, Oxford, 2004).
[31] S. C. Zhang, T. H. Hansson, and S. Kivelson, Phys. Rev. Lett. **62**, 82 (1989).
[32] X.-G. Wen and Y.-S. Wu, Phys. Rev. Lett. **70**, 1501 (1993).
[33] W. Chen, M. P. A. Fisher, and Y.-S. Wu, Phys. Rev. B **48**, 13749 (1993).
[34] M. Barkeshli and J. McGreevy, Phys. Rev. B **89**, 235116 (2014).
[35] F. Jelezko, T. Gaebel, I. Popa, A. Gruber, and J. Wrachtrup, Phys. Rev. Lett. **92**, 076401 (2004).
[36] L. Childress, M. V. Gurudev Dutt, J. M. Taylor, A. S. Zibrov, F. Jelezko, J. Wrachtrup, P. R. Hemmer, and M. D. Lukin, Science **314**, 281 (2006).
[37] M. V. G. Dutt, L. Childress, L. Jiang, E. Togan, J. Maze, F. Jelezko, A. S. Zibrov, P. R. Hemmer, and M. D. Lukin, Science **316**, 1312 (2007).
[38] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.115.026802 for methods and theoretical derivations.
[39] E. Togan *et al.*, Nature (London) **466**, 730 (2010).
[40] We use the symbol σ_{xy} for the Berry curvature even though it only agrees with the Hall conductance in the gapped phases.
[41] E. Lieb, T. Schultz, and D. Mattis, Ann. Phys. (N.Y.) **16**, 407 (1961).
[42] M. B. Hastings, Phys. Rev. B **69**, 104431 (2004).
[43] L. Balents, L. Bartosch, A. Burkov, S. Sachdev, and K. Sengupta, Phys. Rev. B **71**, 144508 (2005).
[44] C. Lannert, M. P. A. Fisher, and T. Senthil, Phys. Rev. B **63**, 134510 (2001).
[45] C. Dasgupta and B. I. Halperin, Phys. Rev. Lett. **47**, 1556 (1981).
[46] M. P. A. Fisher and D. H. Lee, Phys. Rev. B **39**, 2756 (1989).
[47] M. Barkeshli, arXiv:1307.8194.
[48] T. Senthil and M. Levin, Phys. Rev. Lett. **110**, 046801 (2013).
[49] Y.-M. Lu and D.-H. Lee, Phys. Rev. B **89**, 195143 (2014).
[50] T. Grover and A. Vishwanath, Phys. Rev. B **87**, 045129 (2013).
[51] M. Barkeshli and J. McGreevy, Phys. Rev. B **86**, 075136 (2012).
[52] We note that the original vortex fields ϕ^v carry incommensurate momenta at k_0, so their condensation could produce incommensurate ordering at momenta $2k_0$. Identifying such incommensurate orders here requires knowledge of the correlators of the monopole operators M_α, a task beyond the scope of this Letter.
[53] D. Peter, N. Y. Yao, N. Lang, S. D. Huber, M. D. Lukin, and H. P. Büchler, Phys. Rev. A **91**, 053617 (2015).
[54] M. F. Maghrebi, N. Y. Yao, M. Hafezi, T. Pohl, O. Firstenberg, and A. V. Gorshkov, Phys. Rev. A **91**, 033838 (2015).
[55] P. M. Chaikin and T. C. Lubensky, *Principles of Condensed Matter Physics* (Cambridge University Press, Cambridge, England, 2000), Vol. 1.